HW 1-6 page 1

2.5 p 150   47* 49 53* 55 57 63

2.6 p 157   3 − 19 odd, 34

$\boxed{2.5}$ (47) $\tan y = x$

$$\sec^2 y\left(\frac{dy}{dx}\right) = 1\,\frac{dx}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

tan is not defined for odd multiples of $\frac{\pi}{2}$

so differentiable for $\frac{-\pi}{2} < y < \frac{\pi}{2}$

Recall $\sin^2 x + \cos^2 x = 1$

then $\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

so $\tan^2 x + 1 = \sec^2 x$

(49) second derivative at

$$x^2 + y^2 = 4$$

$$2x + 2y\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\left(\frac{dy}{dx}\right)}{y^2}$$

$$= \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$$

$$= -\frac{1}{y} - \frac{x^2}{y^3}$$

$$= \frac{(-1)(y^2+x^2)}{y^3} = \frac{-4}{y^3}$$

(53) 2nd deriv.

$$7xy + \sin x = 2$$

$$7x\frac{dy}{dx} + 7y + \cos x = 0$$

$$\frac{dy}{dx} = \frac{-7y - \cos x}{7x}$$

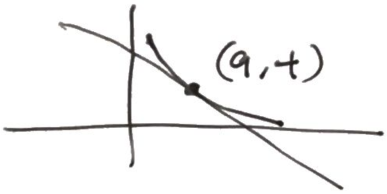
$$\frac{d^2y}{dx^2} = \frac{(7x)\left(-7\left(\frac{dy}{dx}\right) + \sin x\right) - (7)(-7y - \cos x)}{(7x)^2}$$

$$= \frac{-49x\left(\frac{-7y-\cos x}{7x}\right) + 7x\sin x + 49y + 7\cos x}{49x^2}$$

$$= \frac{7(7y - \cos x) + 7x\sin x + 7(7y - \cos x)}{49x^2}$$

$$= \frac{14y + 2\cos x + x\sin x}{7x^2}$$

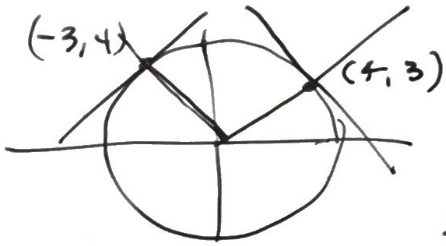
(55) $\sqrt{x} + \sqrt{y} = 5$ @ $(9,4)$ using calculator



from calc $\left.\frac{dy}{dx}\right|_{x=9} = \frac{-2}{3}$

$y - 4 = -\frac{2}{3}(x-9)$

(57)



$x^2 + y^2 = 25$

$2x + 2y\,y' = 0$

$y' = \frac{-x}{y}$

tang. to $(4,3)$ is: $y - 3 = \frac{-4}{3}(x-4)$

norm to $(4,3)$ is: $y - 3 = \frac{3}{4}(x-4$

tang to $(-3,4)$ is: $y - 4 = \frac{3}{4}(x+3)$

norm to $(-3\,4)$ is: $y - 4 = -\frac{4}{3}(x+3$

(just solve for y to type in calculator $Y_1 = \sqrt{25 - x^2}$

$Y_2 = - Y_1$

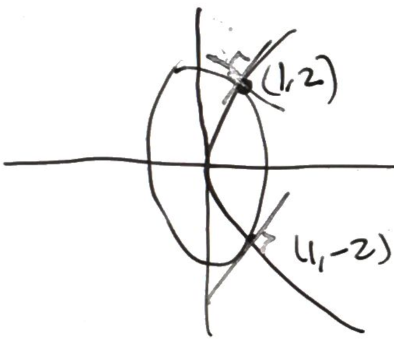
$Y_3 = -4/3(x-4)+3$

⋮

etc

(63) Plot $2x^2 + y^2 = 6$ w/ calc.

$y^2 = 4x$



Ellipse y': $4x + 2y\,y' = 0$

$y' = \frac{-4x}{2y} = \frac{-2x}{y}$

Parabola y': $2y\,y' = 4$

$y' = \frac{4}{2y} = \frac{2}{y}$

so @ $(1,2)$

ellipse $m = \frac{-2(1)}{2} = -1$ ⊥

parabola $m = \frac{2}{2} = 1$

at $(1,-2)$ ellips $m = \frac{-2(1)}{(-2)} = 1$ ⊥

parabola $m = \frac{2}{-2} = -1$

$\boxed{2.6}$ p157

(3) $y = \sqrt{x}$

(a) $\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$ (when $x=4$ $\frac{dy}{dx} = \frac{1}{4}$ $\frac{dx}{dt} = 3$)

$\frac{dy}{dt} = (\frac{1}{4})(3) = \frac{3}{4}$ (y units/time units)

(b) find $\frac{dx}{dt}$ when $x=25$ given $\frac{dy}{dt} = 2$

$\frac{dx}{dt} = \frac{dx}{dy} \cdot \frac{dy}{dt}$ so find $\frac{dx}{dy} = \left(\frac{1}{2\sqrt{x}}\right)^{-1} = 2\sqrt{x}$

$= 2\sqrt{25} \cdot 2 = 20$ x units / time units

(5) $xy = 4$

(a) find $\frac{dy}{dt}$ when $x = 8$ given $\frac{dx}{dt} = 10$

$y = 4x^{-1}$, so $y' = \frac{-4}{x^2}$, so $\left.\frac{dy}{dx}\right|_{x=8} = \frac{-1}{16}$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{-1}{16} \cdot 10 = -\frac{5}{8}$

(b) find $\frac{dx}{dt}$ when $x=1$ given $\frac{dy}{dt} = -6$

$\frac{dx}{dt} = \frac{dx}{dy} \cdot \frac{dy}{dt}$ need $\frac{dx}{dy}$: $x\,(1)\left(\frac{dy}{dy}\right) + (1)\left(\frac{dx}{dy}\right) y = 0$

$\frac{dx}{dy} = \left.\frac{-x}{y}\right|_{(1,4)} = \frac{-1}{4}$

$\frac{dx}{dt} = \left(\frac{-1}{4}\right)(-6 = \frac{3}{2}$

(7) $y = 2x^2 + 1$ and $\frac{dx}{dt} = 2$ cm/sec find $\frac{dy}{dt}$

$\frac{dy}{dx} = 4x$

(a) $x = -1$, $\frac{dy}{dx} = -4$ $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (-4)(2) = -8$ cm/sec

(b) $x = 0$, $\frac{dy}{dx} = 0$ $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = 0 \cdot 2 = 0$ cm/sec

(c) $x = 1$ $\frac{dy}{dx} = 4$ $\frac{dy}{dt} = (4)(2) = 8$ cm/sec

(9) $y = \tan x \qquad \frac{dx}{dt} = 3$ ft/sec find $\frac{dy}{dt}$

$y' = \sec^2 x$

@ $x = \frac{-\pi}{3}$ so $\frac{dy}{dx} = \left(\frac{1}{\cos\frac{-\pi}{3}}\right)^2 = 2^2 = 4$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = 4 \cdot 3 = 12$ ft/sec

(b) $x = \frac{-\pi}{4} \qquad \frac{dy}{dx} = (\sqrt{2})^2 = 2 \qquad \frac{dy}{dt} = (2)(3) = 6$ ft/sec

(c) $x = 0 \qquad \frac{dy}{dx} = 1$ so $\frac{dy}{dt} = (1)(3) = 3$ ft/sec

(11) 

r is increasing at 4 cm/min so $\frac{dr}{dt} = 4$

Find the rates of change of the area when $r = 37$ cm.

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r \Big|_{r=37} = 74\pi$

$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} = (74\pi)(4) = 296\pi$ cm²/min

(13) 

$\frac{dr}{dt} = 3$ in/min

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$.

(a) when $r = 9$ in $\frac{dV}{dr} = 324\pi$ in³/min

so $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} = (324\pi)(3) = 972\pi$ in³/min

when $r = 36$ in $\frac{dV}{dt} = 4\pi(36)^2(3) = 15{,}552\pi$ in³/min

(b) why so different though $\frac{dr}{dt}$ is constant?
Volume is exponentially in growth

(15) 

$\frac{de}{dt} = 6$ cm/sec

$V = e^3 \qquad \frac{dV}{de}\Big|_{e=2} = 12 \qquad \frac{dV}{de}\Big|_{e=10} = 300$

$\frac{dV}{de} = 3e^2$

(a) $\frac{dV}{dt} = \frac{dV}{de} \cdot \frac{de}{dt} = 12 \cdot 6 = 72$ cm³/sec

(b) $\frac{dV}{dt} = 300 \cdot 6 = 1800$ c/sec

(17)

$\frac{dV}{dt} = 10 \text{ ft}^3/\text{min}$

diamur = 3h $r = \frac{3h}{2}$

Volum $= \frac{\pi}{3}\left(\frac{3h}{2}\right)^2 h$

$V = \frac{3h^3}{4}$; $\frac{dV}{dh} = \frac{9h^2}{4}\pi$

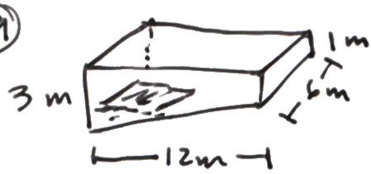
Find $\frac{dh}{dt}$ when $h = 15$

$\frac{dV}{dh}\Big|_{h=15} = \frac{2025\pi}{4}$

$\frac{dh}{dt} = \left(\frac{dh}{dV}\right)\left(\frac{dV}{dt}\right)$

$= \left(\frac{4}{2025\pi}\right)(10) = \frac{8}{405\pi} \text{ ft}^3/\text{min}$

(19)



flow $\frac{1}{4}$ m³/min

1 m of water at the deep end of the pool.

a. What % of the pool is filled?

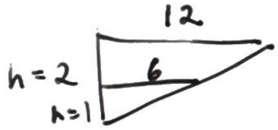
b. Find the rate the water level is rising.

(a) Volum of Pool is a prism w/ trapezoid base

$V = h(BA) = 6\left(\frac{1}{2}(1+3)(12)\right) = 144 \text{ m}^3$

Water is triangular prism

$V = h(BA) = 6\left(\frac{1}{2}(1)(6)\right) = 18 \text{ m}^3$



so $\frac{18}{144} = \frac{9}{72} = \frac{3}{24} = \frac{1}{8}$ (or 12½%)

has water in it.

(b) In general $V = 6\left(\frac{1}{2}(h)(6h)\right) = 18h^2$

so $\frac{dV}{dh} = 36h$ m³/m of h

$\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt} = \frac{1}{36h} \cdot \frac{1}{4} = \frac{1}{144h}$ m/min

for $0 < h < 2$

so since $h = 1$, the rate is $\frac{1}{144}$ m/min.

2.6

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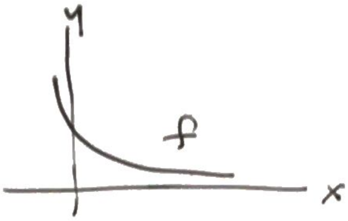
(i)



(ii)

(a.) If $\frac{dx}{dt} < 0$

what of $\frac{dy}{dt}$?

we know

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

in (i) $\frac{dy}{dx} < 0$ so $\frac{dy}{dt} > 0$

in (ii) $\frac{dy}{dx} > 0$ so $\frac{dy}{dt} < 0$

(b) If $\frac{dy}{dt} > 0$

then what of $\frac{dx}{dt}$?

$$\frac{dx}{dt} = \frac{dx}{dy} \cdot \frac{dy}{dt}$$

$$= \left(\frac{dy}{dx}\right)^{-1} \left(\frac{dy}{dt}\right)$$

in (i) $\frac{dy}{dx} < 0$ so $\frac{dx}{dt} < 0$

in (ii) $\frac{dy}{dx} > 0$ so $\frac{dx}{dt} > 0$